Exercise 8

Solve the differential equation.

$$y = y''$$

Solution

This is a linear homogeneous ODE with constant coefficients, so its solutions are of the form $y = e^{rx}$.

$$y = e^{rx} \quad \rightarrow \quad y' = re^{rx} \quad \rightarrow \quad y'' = r^2 e^{rx}$$

Plug these formulas into the ODE.

$$e^{rx} = r^2 e^{rx}$$

Divide both sides by e^{rx}.

$$1 = r^2$$

Solve for r.

$$r = \{\pm 1\}$$

Two solutions to the ODE are e^{-x} and e^x. By the principle of superposition, then,

$$y(x) = C_1 e^{-x} + C_2 e^x,$$

where C_1 and C_2 are arbitrary constants.